On the Science of Speed Scaling From an Algorithmist's Viewpoint

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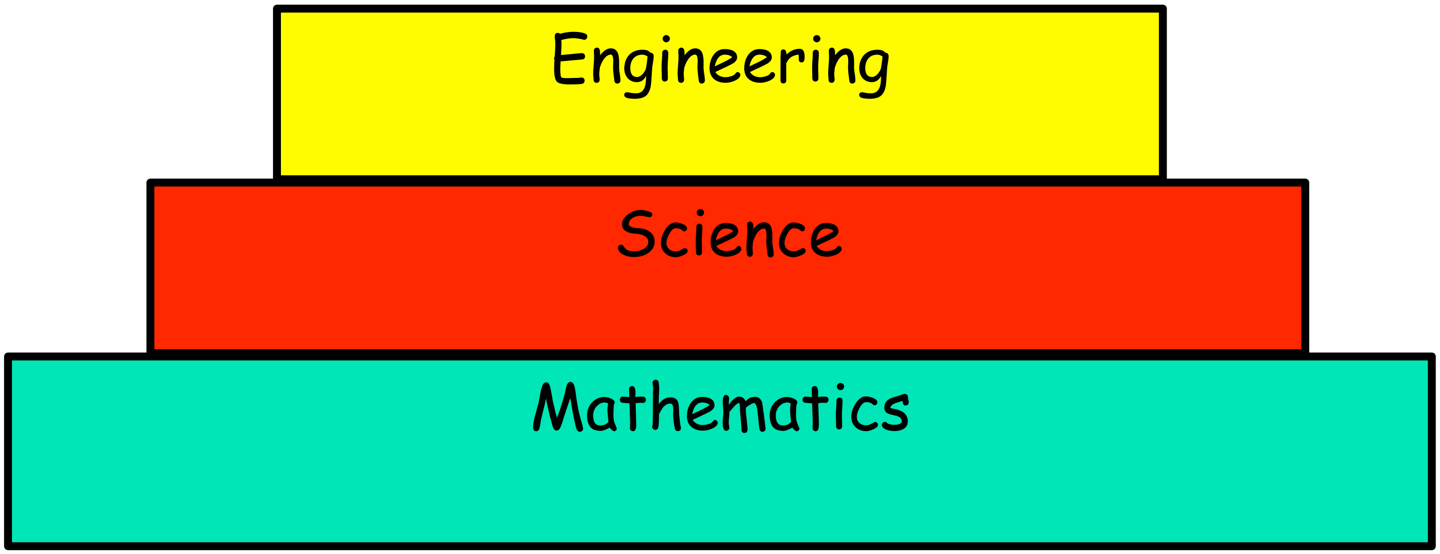
Science's Place in the World



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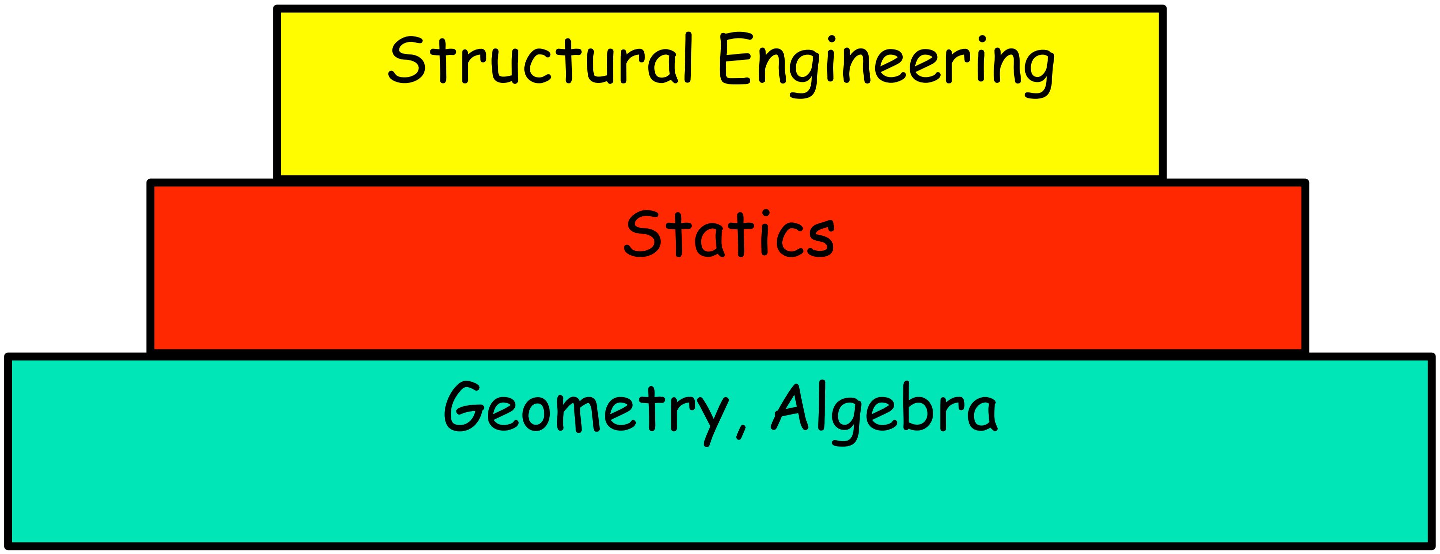
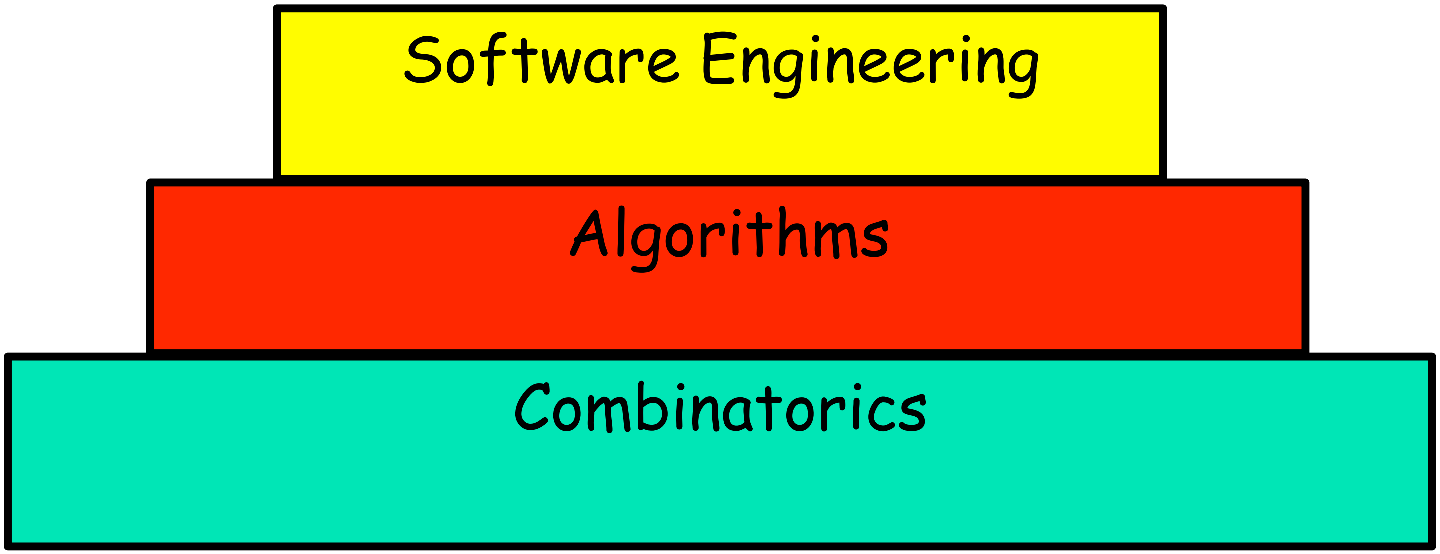
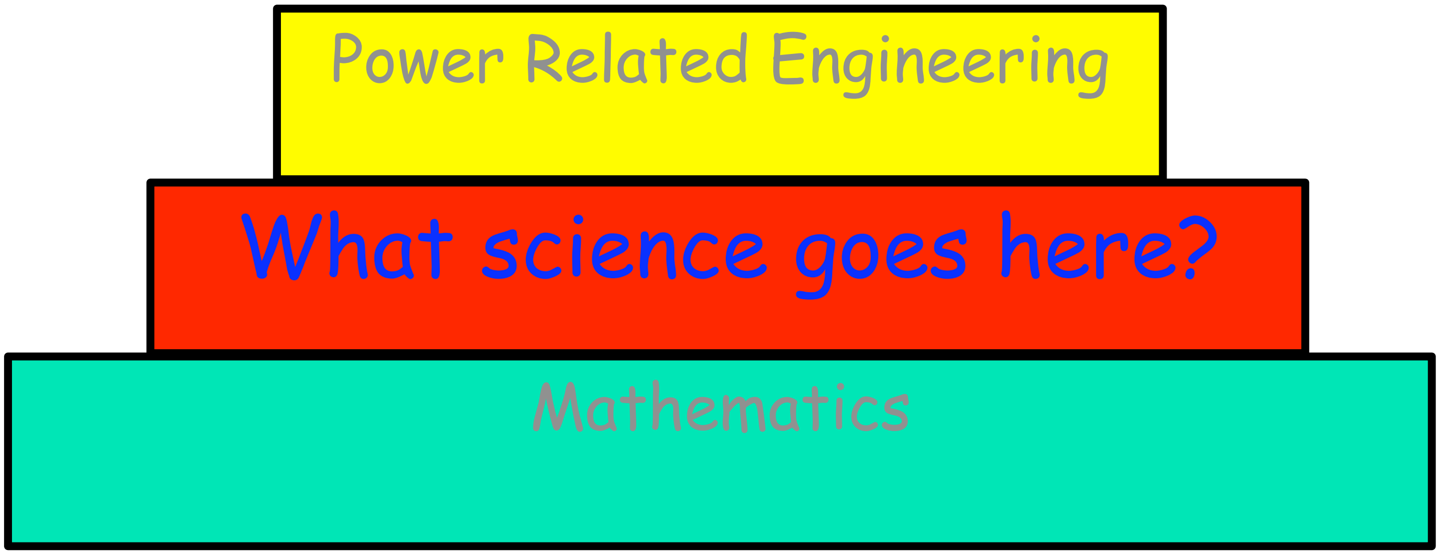


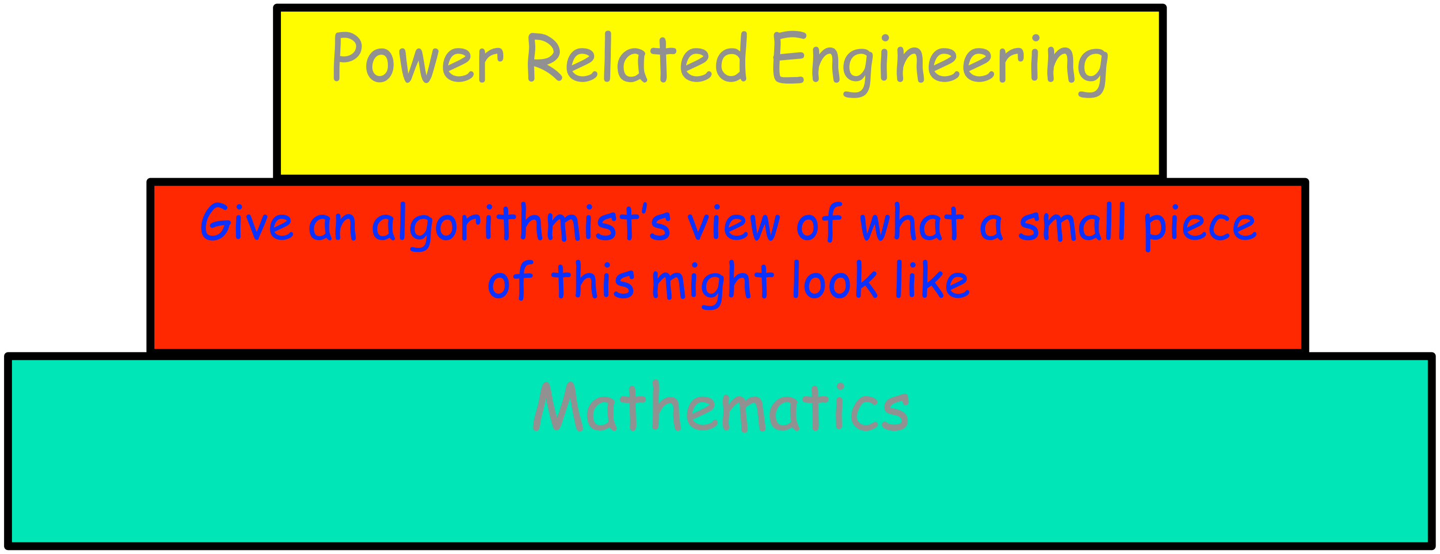
Science's Place in the World



Workshop's Goal: Give Some Initial Answer

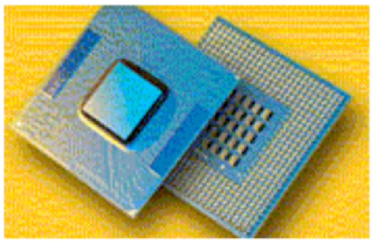


This Talk's Goal



Motivating Technology: Speed Scaling

Mobile Intel® Pentium® 4 Processor - M



Built on 0.13-micron process technology and Intel® NetBurst™ microarchitecture, the Mobile Intel® Pentium® 4 Processor - M provides innovative capabilities for graphics-intensive multimedia applications. It's also excellent for processor-intensive background computing tasks, such as compression, encryption, and virus scanning.

Enhanced Intel SpeedStep® technology helps to optimize application performance and power consumption, and Deeper Sleep Alert State, a dynamic power management mode, adjusts voltage during brief periods of inactivity—even between keystrokes—for longer battery life.

Mobile Intel® Pentium® 4 Processor – M Features

Available Speeds	**2.60 GHz**, 2.50 GHz, 2.40 GHz, 2.20 GHz, 2.0 GHz, 1.80 GHz, 1.70 GHz, 1.60 GHz, 1.50 GHz, 1.40 GHz
Chipset	Mobile Intel® 845 Chipset Family
Cache	512 KB On-Die Level 2 (L2) Cache
RAM	up to 1GB DDR SDRAM
System Frequency Bus	400 MHz

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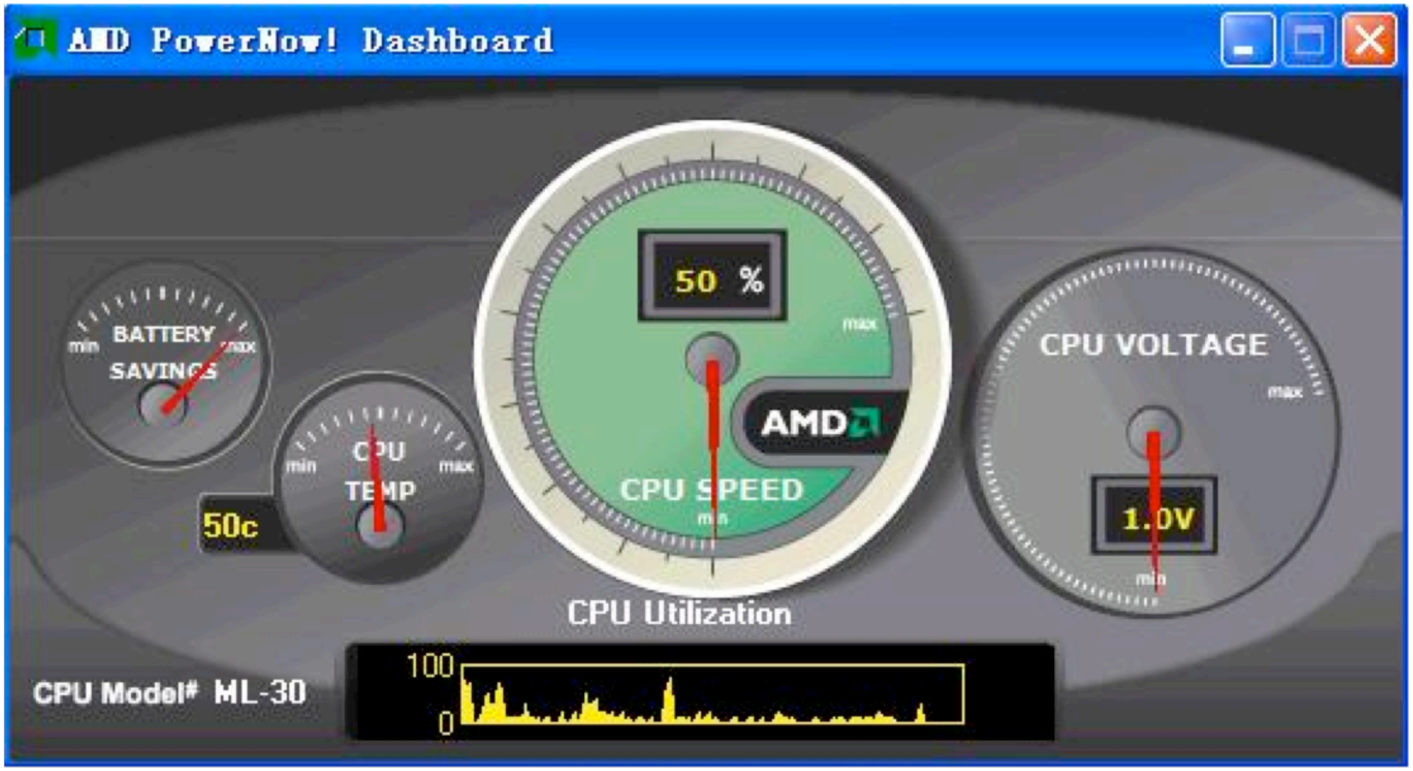
Motivating Technology: Speed Scaling



One Question Raised By This Technology

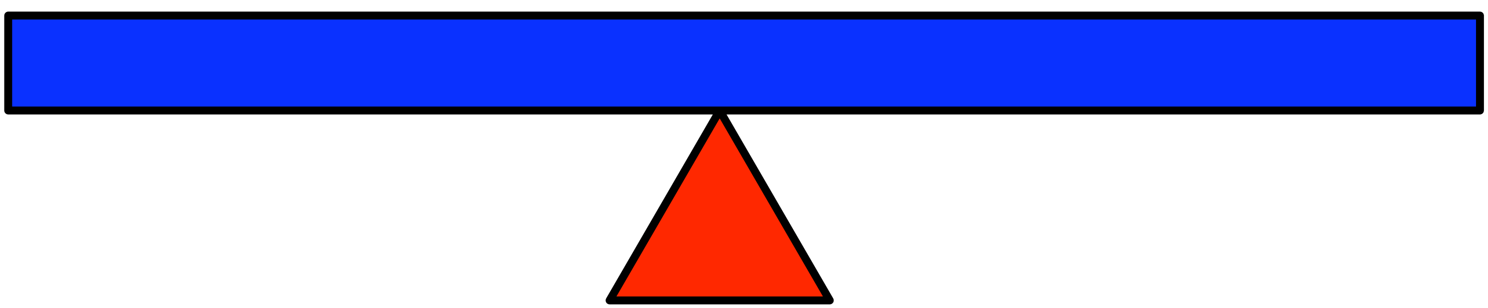
- Engineering Question: How do you manage/ scale the speed/power, and how does this interact with scheduling?
  - Overarching engineering question for this talk

Science from the Algorithmist's View

- Science research tries to model a complex system by something simple, accurate, amenable to math and predictive.
  - Google researcher Muthu Muthukrishnan's "My Slice of Pizza" blog

- Accurate
- Realistic
- Predictive

- Simple
- Amenable to math



Another Muthu Quote:

- I realized this week in a meeting (at Google) that we have people who see the tree and those that see the forest, neither is useful. We need people who see a tree and know it is in a forest, and who see a forest and know that it is made up of trees.

Agenda

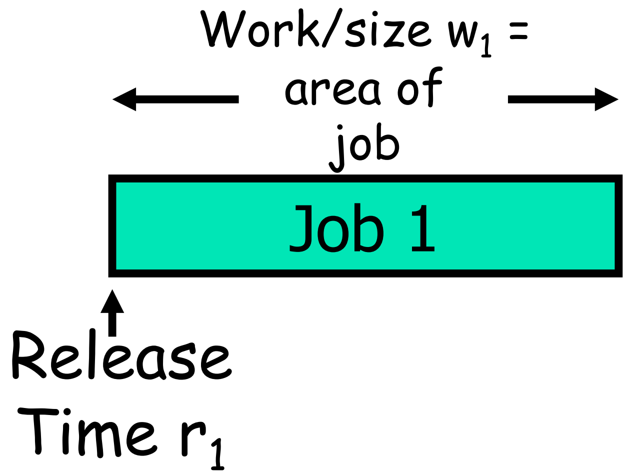
- Forest: How do you manage/scale the speed/power, and how does this interact with scheduling?
- Enumerate some of the interesting trees/models in the forest:
  - Physics
  - Objective
  - Analysis method
  - etc.
- Explain what is known about a couple interesting trees, and relate this back to the forest

Modeling Policies/Algorithms and Schedules

- The system needs
  - Job selection algorithm: determines which job is run
    - e.g. Shortest Job First,
    - Shortest Remaining Processing Time,
    - Shortest Elapsed Time, etc.
  - Speed scaling algorithm: determines the speed that the processor is run

Modeling Policies/Algorithms and Schedules

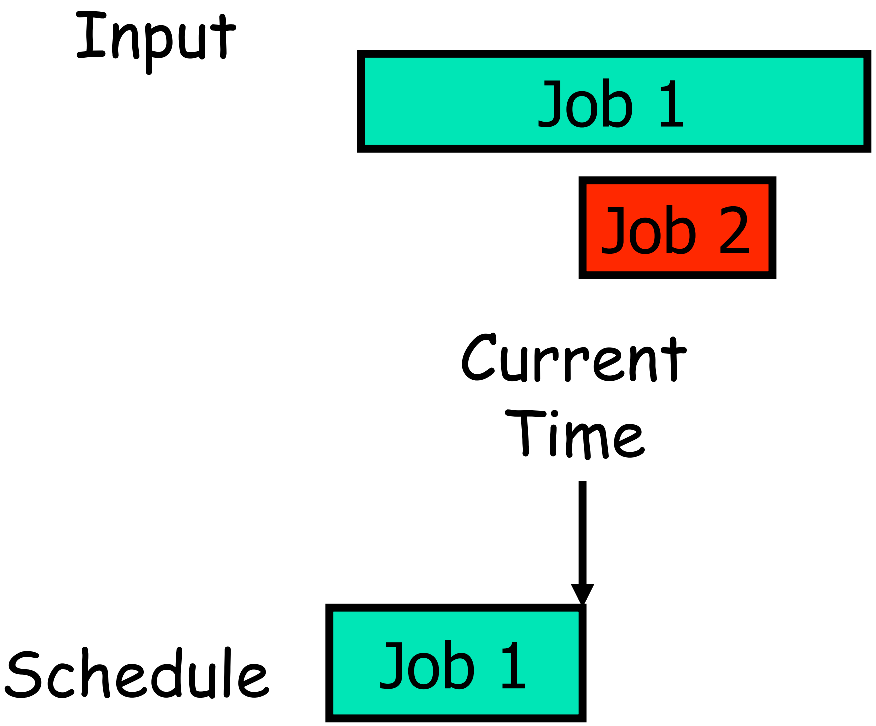
Input



Schedule

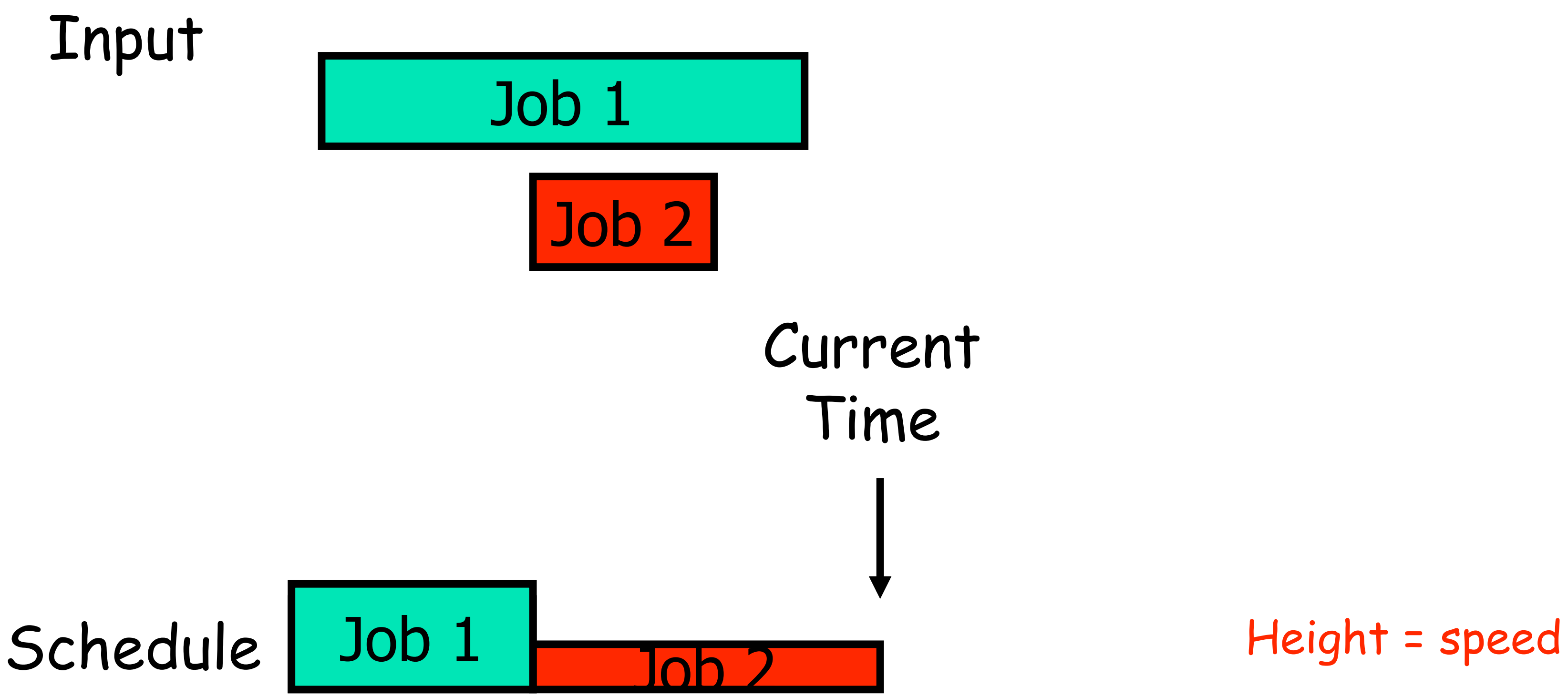
Height = speed

Modeling Policies/Algorithms and Schedules

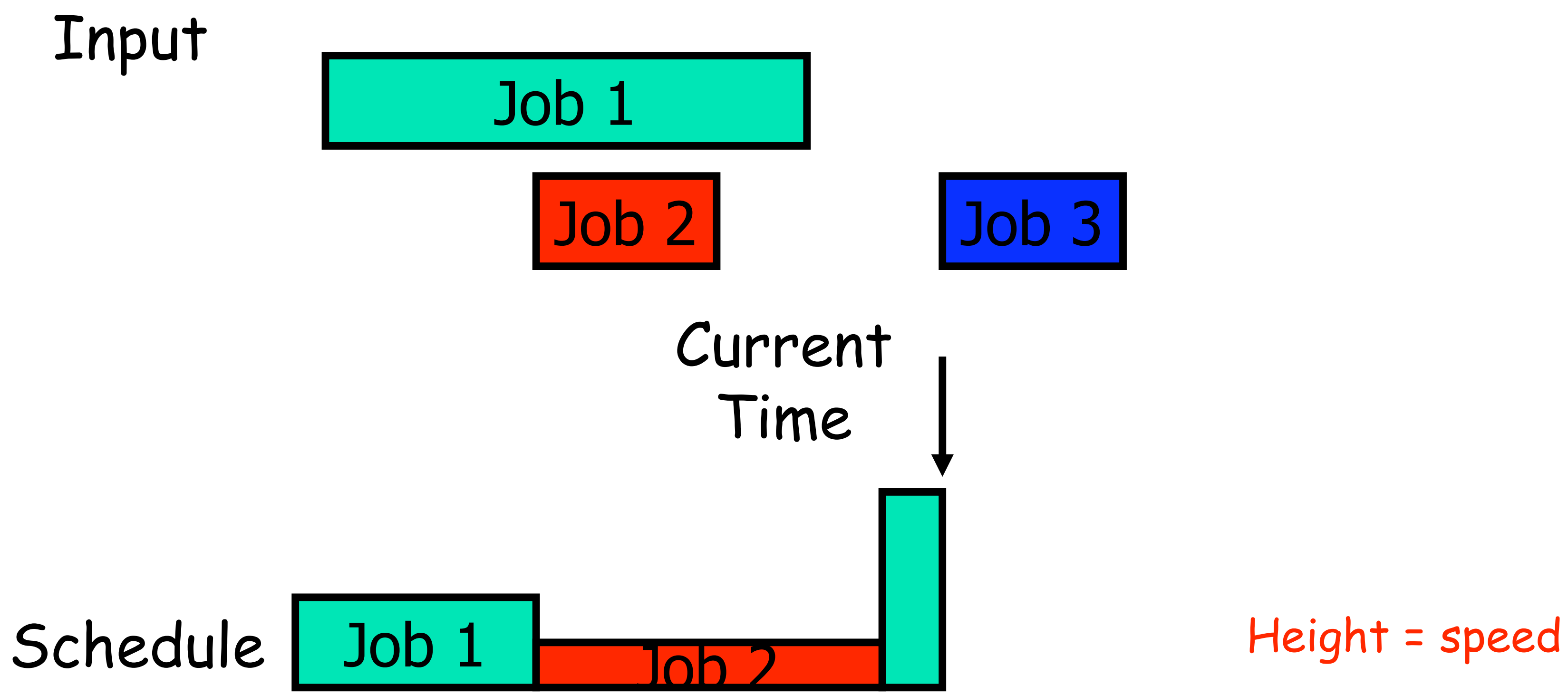


Height = speed

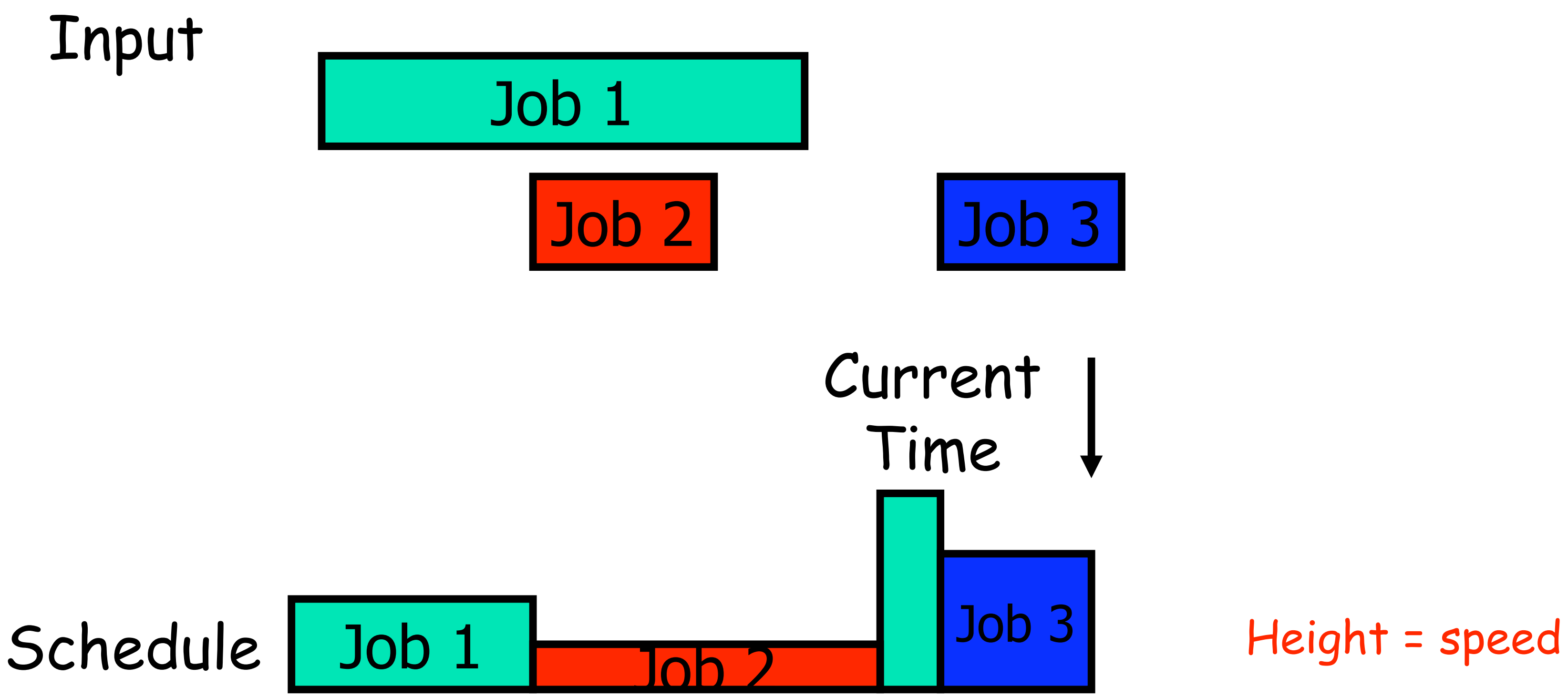
Modeling Policies/Algorithms and Schedules



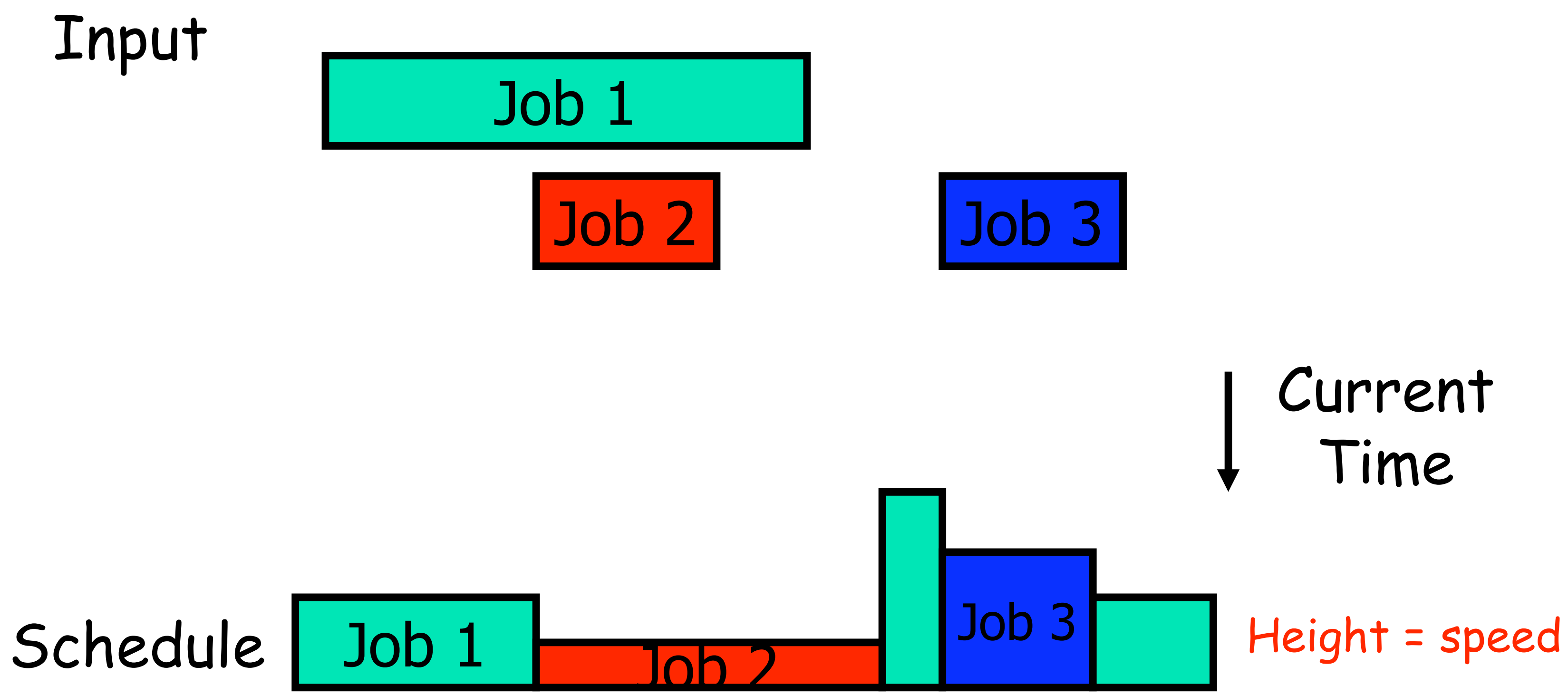
Modeling Policies/Algorithms and Schedules



Modeling Policies/Algorithms and Schedules

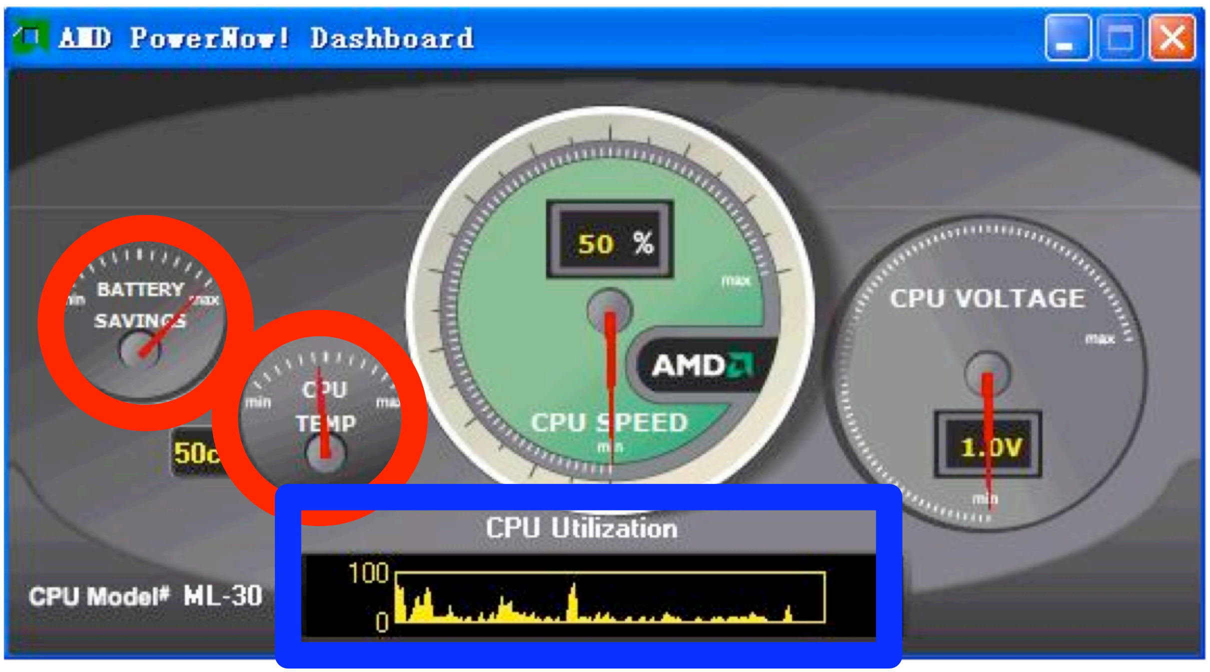


Modeling Policies/Algorithms and Schedules



Scheduling Objectives

- Dual Objectives:
  - Some Quality of Service (QoS) measure of the schedule
    - e.g. deadline feasibility, average response time, worst-case response time, average slow down, etc.
  - Some power related objective
    - e.g. temperature, energy



Physical Models

- Allowable Speeds
  - Continuous and unbounded
  - Continuous and bounded
  - Discrete
- Power P as a function of speed s
  - $P = s^{\alpha}$ where α is some constant
    - Motivated by cube-root rule for dynamic power in CMOS based processors, i.e. $\alpha \approx 3$
  - $P = s^{\alpha}$ + constant static power
  - P = f(s) for some arbitrary function f

Physical Models

- Energy = $\int_{time}$ Power
- Temperature T
  - Newton's Law of Cooling: rate of heat loss of a body is proportional to the difference in temperatures between the body and its surroundings
    - $dT/dt = P - b\,T$
    - b is device dependent constant

How to Compare Algorithms?

- Average Case (Queuing Theory)
  - Assume a mathematically tractable input distribution, e.g. Poisson arrivals and exponential job sizes, and compute expected performance of the algorithms
- Worst-case Relative Error (Competitive Analysis)
  - An algorithm *A* is competitive if it has bounded relative error
    - $\max_I |A(I) - Optimal(I)|/Optimal(I) < \infty$
  - An algorithm is optimally competitive if it has minimal worst-case relative error among all possible algorithms

Agenda

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Cool Tree 1: Deadline Feasibility and Temperature

- Assume that somehow a system knows a deadline for each job
  - This makes the scheduling QoS objective a constraint
  - WLOG one can use Earliest Deadline First for the job selection algorithm
  - Allows one to focus on the speed scaling algorithm
- Power objective is to minimize the maximum temperature ever reached by the device.

Cool Tree 1: Deadline Feasibility and Temperature

- Understanding Newton's Law:
  $dT/dt = P - bT$
  - If b=0, max temperature = energy
  - If $b=\infty$, max temperature = max power
  - Key theorem for analysis: Maximum temperature ≈ maximum energy over any time interval of length 1/b

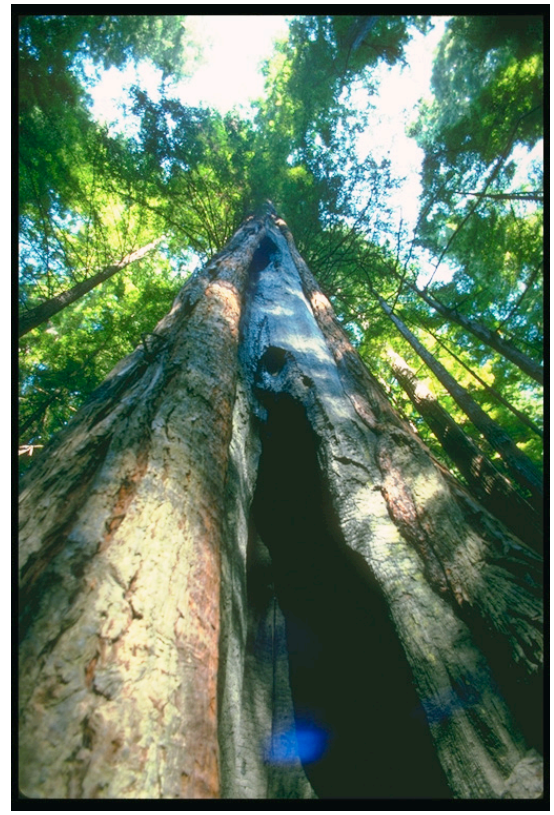
- Natural Question: Does the speed scaling algorithm require knowledge of the device specific cooling parameter b to be competitive?
  - Assume power = polynomial in speed

Cool Tree 1: Deadline Feasibility and Temperature

- Theorem: There is an optimally competitive algorithm when $b=\infty$
  - Algorithm Description: At time t, run at speed equal to the maximum over all $t_1 < t < t_2$ of $e^*w(t_1, t, t_2)$
    - Where $w(t_1, t, t_2)$ is the aggregate size of the jobs that arrive after t_1, and before t, with deadlines before t_2
  - Note that this algorithm is reasonably simple, but sufficiently non-intuitive that it is hard to imagine it being discovered by experimentation and local search
- Theorem: This algorithm is simultaneously competitive for all cooling parameters b
  - Some natural algorithms that are competitive for energy (b=0), are not competitive for larger b

Cool Tree 2: Energy/Response Tradeoff

- Setup: A user specifies an energy amount ρ that he/she is willing to spend to get a unit improvement in response time
  - e.g. I am willing to spend 3 ergs of energy for a 1 microsecond improvement in response time for a particular job
  - Response time of a job = completion time minus release time
- Resulting Objective:
  ρ *total response time + energy used



Cool Tree 2: Energy/Response Tradeoff

- Natural Question: As ρ decreases, and the total energy used decreases, can the energy of particular jobs increase in the optimal schedule ?
  - Recall ρ = amount of energy that you are willing to spend to get a unit decrease in response time

Cool Tree 2: Energy/Response Tradeoff

- Natural Question: As ρ decreases, and energy used decreases, can the energy of particular jobs increase in the optimal schedule ?
- Theorem: In the optimal schedule the power of a job J is proportional to the number of jobs that would be delayed if J is delayed (Modulo special cases)
- Answer: So as energy is lost, and jobs interfere more, this theorem forces the energy for some jobs up at a faster rate than energy is lost

- The optimal schedule doesn't change smoothly as a function of energy.
- Open algorithmic problem: find an efficient algorithm to find the optimal schedule, or proof that no such algorithm exists

Cool Tree 2: Energy/Response Tradeoff

- Natural Question: Do properties of the power function affect whether one can have a competitive algorithm?
  - Equivalently, can one reason about arbitrary power functions?

Cool Tree 2: Energy/Response Tradeoff

- Natural Question: Do properties of the power function affect whether one can have a competitive algorithm?
- One answer: If the algorithm doesn't know the size of a job when it arrives, then the algorithm can not be competitive if the power function is very steep

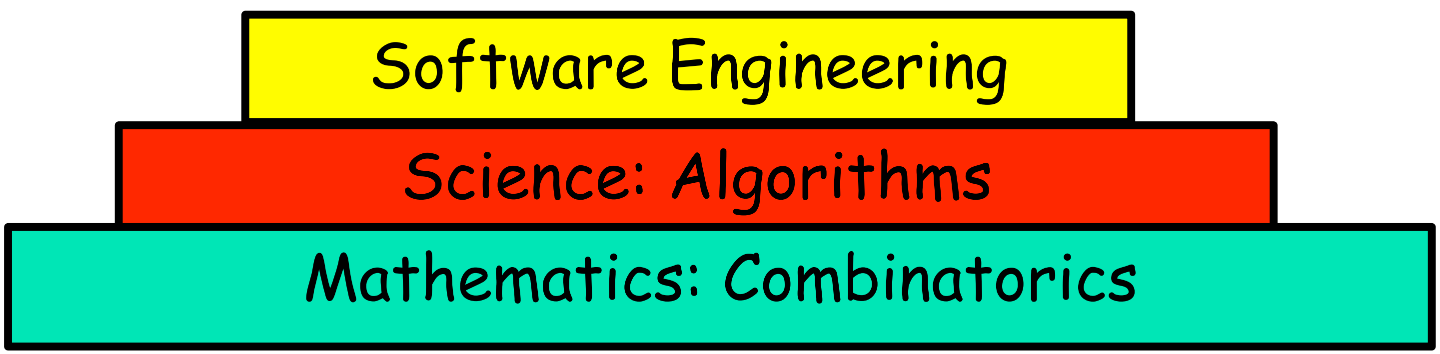
Cool Tree 2: Energy/Response Tradeoff

- Natural Question: Do properties of the power function affect whether one can have a competitive algorithm?
- Another answer: The following algorithm is competitive for all power functions:
  - Job selection: Shortest Remaining Processing Time (SRPT)
  - Speed Scaling: Power = number of unfinished jobs + 1
- The analysis requires completely different techniques than the analysis of SRPT with a fixed speed processor because
  - With a fixed speed processor the resource available per unit time is fixed
  - With a variable speed processor, the resource (energy) is global, and you get a concave nonlinear return for investing resources at a particular time

Some Concluding Remarks About the Forest:

Predictability of the Model

- Muthu's Definition: Science research tries to "model" a complex system by something simple, accurate, amenable to math and predictive.
  - In practice, predictive usually means that thinking abstractly/ mathematically within the context of the model is useful for finding a good engineering solution, not that the model perfectly predicts what will happen in practice
- Google wants software engineers that have algorithms training, not because the RAM model perfectly predicts the performance of an algorithm on a real computer, but because being able to reason about computation in an abstract model is useful when searching for a solution on a real computer

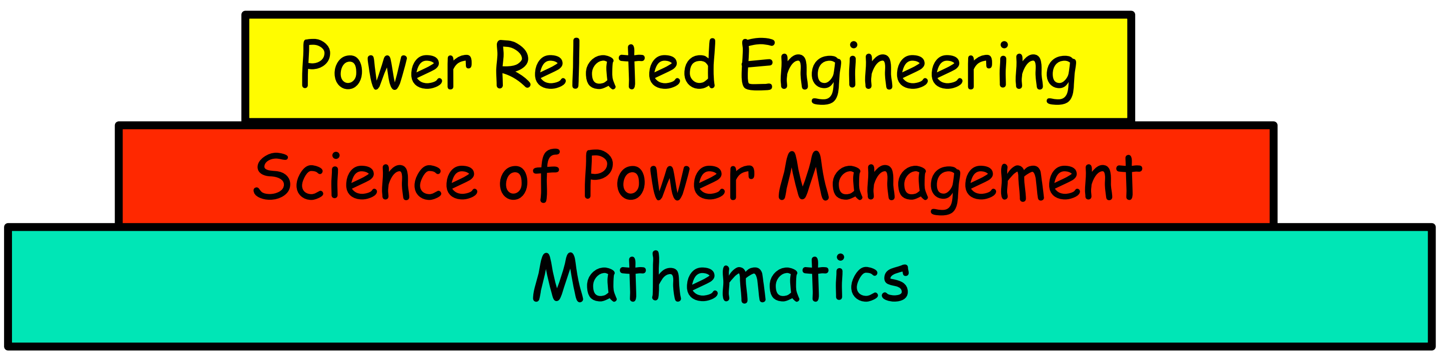


What Was Learned From This Research

- We got way better at reasoning about energy and temperature as resources, at least within the types of models considered
  - Allowable Speeds
    - Best Model: Continuous and unbounded. Other models are more complicated without providing more insight
  - Power P as a function of speed s
    - For some problems, one can reason about an arbitrary power function
  - Newton's Law of Cooling $dT/dt = P - bT$
    - The easiest way to think about approximate temperature is energy over time intervals of length $1/b$
- I can plausibly imagine teaching future engineers to think abstractly about power/energy/temperature as we currently teaching future software engineers to think about time/space.

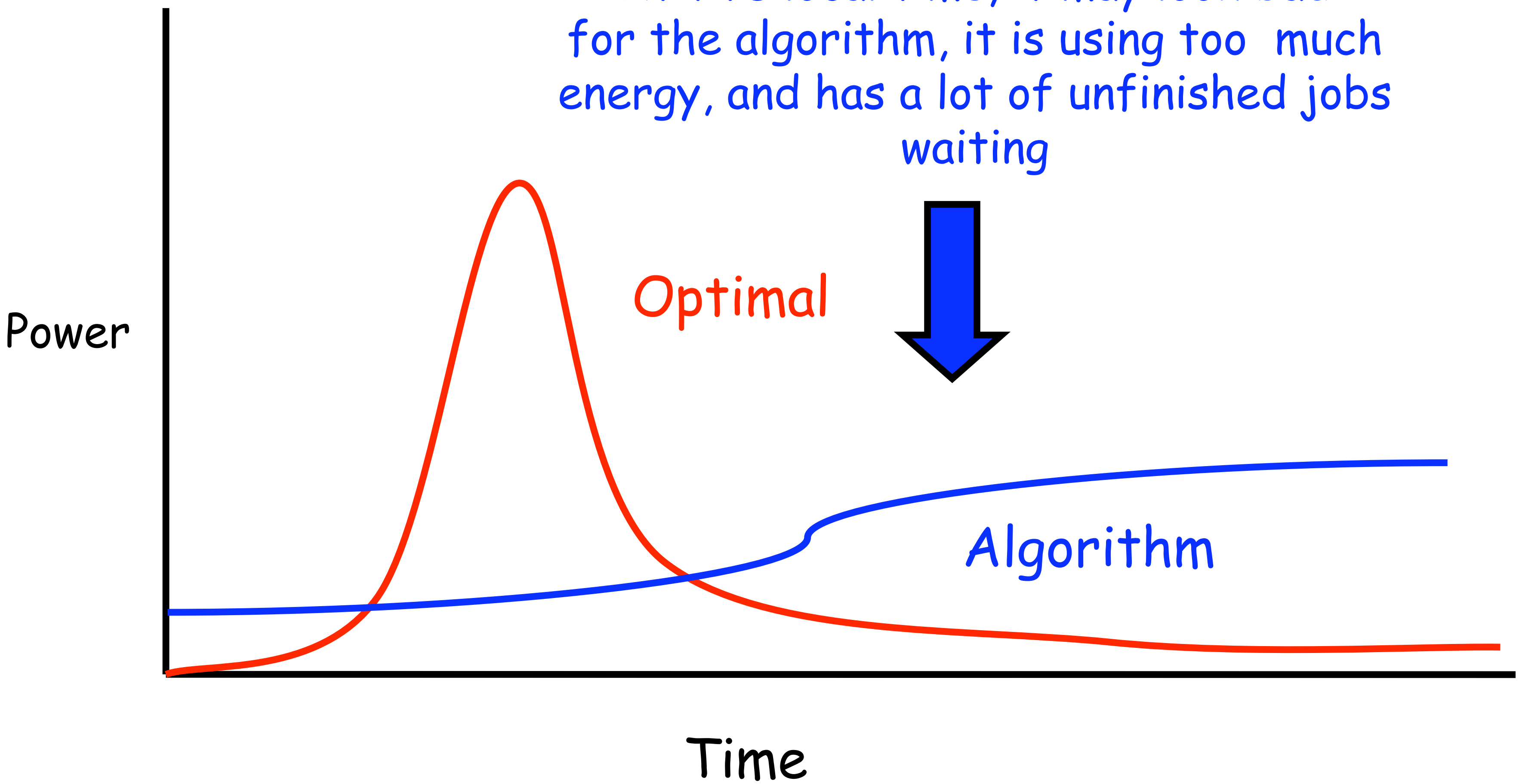
Science of Power Management

- Questions for this workshop:
  - What might a science of power management look like?
  - What areas of power management are most amenable to scientific investigation?
  - What areas of power managements would most benefit from a better scientific foundation?
  - What benefits are there for a better scientific foundation for these areas of power management?



Speed scaling analysis is hard because energy is a global resource that gives nonlinear returns from investment:

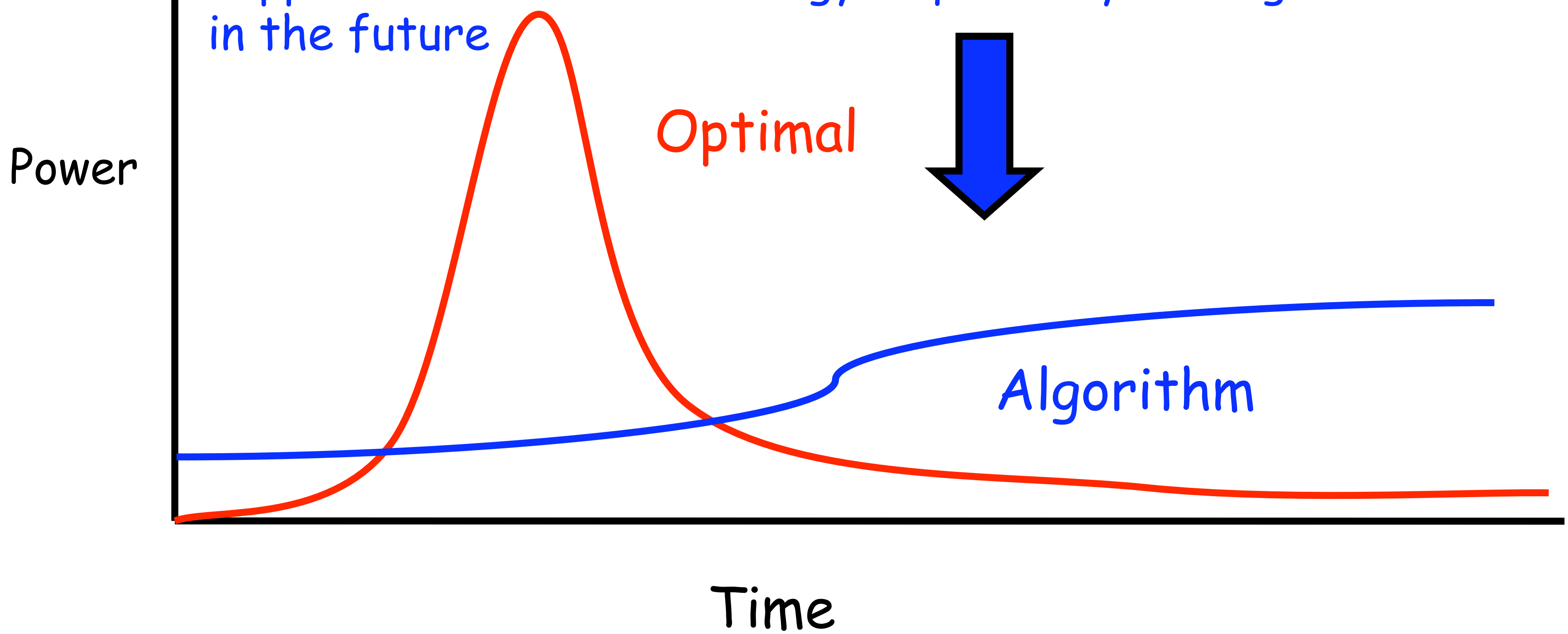
At this local time, it may look bad for the algorithm, it is using too much energy, and has a lot of unfinished jobs waiting



Speed scaling analysis is hard because energy is a global resource:

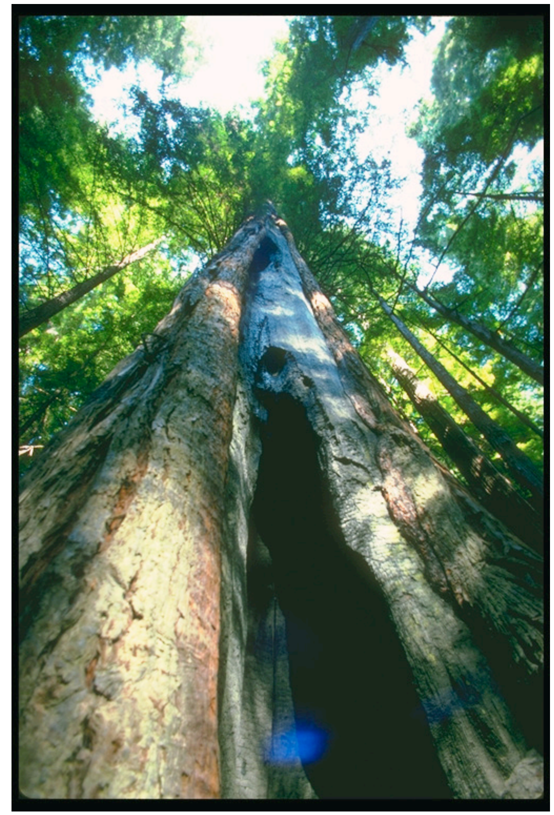
Main Analysis technique: Define a potential function Φ that:

- Lower bound on energy savings of the algorithm in the past
- Upper bound on extra energy required by the algorithm in the future



Cool Tree 2: Energy/Response Tradeoff

- If all jobs are the same size and Power=speed$^{\alpha}$:
  - the potential function Φ = (number of excess jobs)$^{2-1/\alpha}$ works
  - that is, Φ is an lower bound on energy savings in the past, and an upper bound on the extra energy required in the future



Cool Tree 2: Energy/Response Tradeoff

- For arbitrary jobs sizes and an arbitrary power function P, Φ is more complicated:
  - $\Phi = \int_{\text{job sizes } q} f(\text{number of excess jobs of size at least } q)$
  - Where $f(x) = f(x-1) + P'(P-1(x))$